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Optimal design of a thick-walled sandwich pipe

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Abstract—This paper presents a procedure for obtaining an optimal design for sandwich composite pipes. Two designs are considered, namely the optimal design of flexural stiffness and bending strength with regard to certain weight or cost and that of minimum weight and minimum cost with regard to certain flexural stiffness or bending strength. There are many design variables to consider when structuring a sandwich composite pipe. To simplify the design parameters, therefore, a set of non-dimensional variables is presented as design parameters for the optimization of sandwich pipe structures. In this paper, the influence of the design parameters on the flexural rigidity and bending strength of sandwich pipes is discussed with reference to such non-dimensional design parameters. Reduced weight and cost can be achieved by carefully selecting the parameters and geometry of a sandwich pipe.

Keywords: Optimal design; sandwich cylindrical pipe; bending; design of composite structures.

1. INTRODUCTION

In recent years, the use of composites has undergone substantial growth in many fields, such as the sporting goods industry, the automotive industry, airplane construction, highway bridges, and the transportation system [1–5]. The development of sandwich-type structural elements for many industrial products utilizing composites has already received considerable attention. Sandwich composite structures consist of two skin layers and a core layer. By using different materials for the skins and core — for example, metal or laminated composite materials for the outer layers and an non-reinforced material for the core layer — it is possible to achieve quite specific end-product properties compared with either conventional or advanced composites [2, 3, 6]. Moreover, sandwich constructions can be suitably tailored to make effective use of each material property in the molding parts. The most crucial factors are usually aspects of balanced strength and stiffness, balanced thermal property, reduced weight, or various combinations of these.

Various investigations for the optimization of laminated composite plates have been carried out [7–12]. Abrate [7] reported the optimum design of laminated plates and shells subjected to constraints on strength, stiffness, and buckling loads. Adali *et al.* [8] gave optimal weight and deflection designs of thick laminated sandwich plates based on a high-order theory. Based on genetic algorithm analysis, optimal stacking sequence designs of composite plates were studied by Sivakumar *et al.* [9] and Kim *et al.* [10]. Tauchert and Adibhatla [11] optimized the design of symmetrically laminated plates using minimum strain energy as the optimality criterion. Strain energy is taken as the objective function. Morton *et al.* [12] described a procedure for obtaining an optimal minimum area design of a uniform composite I-beam with regard to structure failure. In the optimal analysis, the optimum procedure known as the Complex method was employed to derive the optimal design results.

However, few studies on laminated composite tubes have been reported. Seibi and Amateau [13] developed a design methodology based on a sensitivity analysis. The optimization for controlling the residual thermal stresses of laminated composite tubes was achieved by combining the methodology with a finite-element analysis. On the basis of a genetic algorithm analysis, Jaunky *et al.* developed the analysis and weight optimization strategy for composite grid-stiffened cylinders subjected to global and local buckling constraints and strength constraints [14].

In this paper, we discuss the variation of the mechanical properties, rigidity and strength, as a function of a sandwich geometry with respect to two other important engineering parameters — weight and cost. Moreover, the cost and the weight are also discussed in terms of the engineering properties, rigidity and strength. Because of the many design parameters to be considered in sandwich pipe design, a set of non-dimensional variables is presented as the design parameters for the analysis and optimization of the sandwich pipe structures.

2. OPTIMIZING BENDING PROPERTIES

Two design problems involving the minimization of deflection and maximization of strength are investigated for thick-walled sandwich pipes. The sandwich structure is composed of a core layer and two skin layers of equal thickness separated by the core layer, as shown in Fig. 1. The inner radius of a constant R_0 for the sandwich pipe is designed. The thickness ratios of the skin and core layers are design variables expressed as t_f and t_c, respectively.

2.1. Flexural rigidity

The deformation resistance of a sandwich pipe to bending forces is termed its flexural rigidity (D). The flexural rigidity of the sandwich pipe construction shown in Fig. 1 is given by

$$D = \frac{\pi R_0^4 E_f}{4}\Big\{\Big[\big(1+t_f/R_0\big)^4 - 1 + \big(R/R_0\big)^4 - \big(R/R_0 - t_f/R_0\big)^4\Big] + m\Big[\big(R/R_0 - t_f/R_0\big)^4 - \big(1+t_f/R_0\big)^4\Big]\Big\}, \quad (1)$$

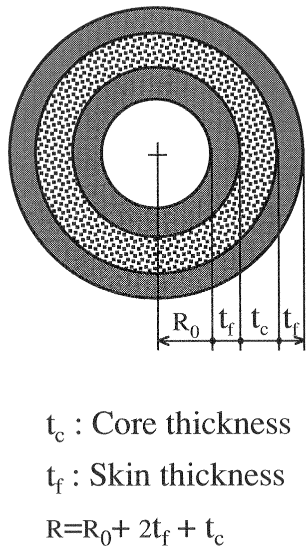

Figure 1. Cross-section of the sandwich pipe.

where $m = E_c/E_f$ is defined as the modulus ratio of the core to the skin layer.

2.2. Bending strength

It is assumed that the core material will not fail under shear stress before tensile failure in the skin materials is attained. Moreover, the ratio of ultimate tensile strength to stress in the skin layers will be less than that in the core layer. Therefore, the failure of the sandwich pipe will result from tensile failure in the skin materials. The maximum bending moment is given by

$$M = [\sigma]\frac{2(\overline{EI})}{E_f(R/R_0 - 1)R_0}, \tag{2}$$

where $\overline{EI}$ is equal to the flexural rigidity D and $[\sigma]$ is ultimate tensile strength.

In order to obtain optimum sandwich constitutions, we consider that the sandwich pipe construction has a certain value with respect to weight or cost.

2.2.1. Optimal design problem with respect to weight. A given constant of weight (W) per unit length can be given by

$$\begin{aligned} W &= \text{skin weight} + \text{core weight} \\ &= \pi\rho_f\left[(R_0 + t_f)^2 - R_0^2 + R^2 - (R - t_f)^2\right] \\ &\quad + \pi\rho_c\left[(R - t_f)^2 - (R_0 + t_f)^2\right]. \end{aligned} \tag{3}$$

$$\text{If } \alpha_f = \pi\rho_f R_0^2/W, \quad \alpha_c = \pi\rho_c R_0^2/W, \quad \text{and } \alpha = \pi\rho_c R^2/W. \tag{4}$$

Substituting these in the expression for the weight relation function, the thickness ratios t_f/R_0 and R_0/R can be given by the above parameters, α_f, α_c, and α, expressed as

$$s = t_f/R_0 = \frac{\alpha_c[1-(\alpha-\alpha_c)]}{2(\alpha_f-\alpha_c)(\alpha-\alpha_c)}\left(\sqrt{\alpha/\alpha_c}-1\right), \tag{5}$$

$$x = R/R_0 = \sqrt{\alpha/\alpha_c}, \tag{6}$$

where α_f and α_c are the non-dimensional parameters expressing the densities of skin and core layers, respectively.

α is a non-dimensional design variable of the outer radius with respect to a given total weight (W). When the skin and the core thickness are equal to zero, respectively, the parameter α reaches a maximum and a minimum. The design range of α is given by

$$\alpha_c(1+1/\alpha_f) \leqslant \alpha \leqslant 1+\alpha_c. \tag{7}$$

The thickness ratio (λ) of skins with respect to a sandwich pipe can be written as

$$\lambda = \frac{2t_f}{R-R_0} = \frac{\alpha_c[1-(\alpha-\alpha_c)]}{(\alpha_f-\alpha_c)(\alpha-\alpha_c)}. \tag{8}$$

The expressions of the relative flexural rigidity and the relative bending strength can be rewritten as follows

$$\frac{4D}{\pi R_0^4 E_f} = \left[(1+s)^4-1+x^4-(x-s)^4\right]+m\left[(x-s)^4-(1+s)^4\right], \tag{9}$$

$$\frac{2M}{\pi R_0^3[\sigma]} = \frac{1}{x-1}\left\{\left[(1+s)^4-1+x^4-(x-s)^4\right]+m\left[(x-s)^4-(1+s)^4\right]\right\}. \tag{10}$$

2.2.2. Optimal design problem with respect to cost. A constant value of the cost (C) per unit length can be given by

$$\begin{aligned} \mathrm{C} =\ & (\text{skin weight}) \times (\text{skin cost/Kg}) + (\text{core weight}) \times (\text{core cost/Kg}) \\ =\ & \pi\rho_f C_f\left[(R_0+t_f)^2-R_0^2+R^2-(R-t_f)^2\right] \\ & +\pi\rho_c C_c\left[(R-t_f)^2-(R_0+t_f)^2\right]. \end{aligned} \tag{11}$$

$$\text{If } \alpha_f = \pi\rho_f C_f R_0^2/C, \quad \alpha_c = \pi\rho_c C_c R_0^2/C, \quad \text{and } \alpha = \pi\rho_c C_c R^2/C. \tag{12}$$

Substituting these in the expression for the cost relation function, the thickness ratios t_f/R_0 (the skin thickness to the inner radius) and R_0/R (the inner to outer radii) can be given by the same formats as those for the optimal design problem with respect to weight.

The parameters, α_f and α_c, are non-dimensional parameters related to the densities and cost of the skin and core layers, respectively.

α is a non-dimensional design variable of the outer radius with respect to a given total weight (C).

3. OPTIMIZING COST OR WEIGHT

The properties of materials are usually closely related to their cost, and the cost is directly associated with weight. Although high-quality products can be produced at a high cost, there exits an optimal cost due to a weight mismatch for two materials in a sandwich construction.

Optimum sandwich pipe construction can be designed by considering the configuration in accordance with the cost ratios of the skin to core materials so as to achieve the same design requirements at the minimum cost. That is, the minimum cost can be obtained on condition that the flexural rigidity or the bending strength holds certain values.

The relative cost of sandwich pipes per unit length shown in equation (11) can be rewritten as

$$\begin{aligned} \frac{C}{\pi\rho_f C_f R_0^2} =& \left[(1+s)^2 - 1 + x^2 - (x-s)^2\right] \\ &+ \zeta\left[(x-s)^2 - (1+s)^2\right], \\ \zeta =& \ \eta\xi, \end{aligned} \tag{13}$$

where $\eta = \rho_c/\rho_f$ and $\xi = C_c/C_f$ are defined by the density ratio and the cost ratios of core to skin layers, respectively.

When parameter $\xi = 1$, equation (13) becomes the optimal problem of minimum weight of the sandwich pipe, rewritten as

$$\frac{W}{\pi\rho_f R_0^2} = \left[(1+s)^2 - 1 + x^2 - (x-s)^2\right] + \eta\left[(x-s)^2 - (1+s)^2\right]. \tag{14}$$

3.1. Optimal design problem with respect to flexural rigidity

Considering the condition of a given flexural rigidity D, equation (1) can be rewritten as

$$D = \frac{\pi R_0^4 E_f}{4}\left[(1+s)^4 - 1 + x^4 - (x-s)^4\right] + m\left[(x-s)^4 - (1+s)^4\right]. \tag{15}$$

Furthermore, the above expression can be written as the following equation form:

$$s^3 + (3/2)(1-x)s^2 + (x^2 - x + 1)s + d = 0, \tag{16}$$

where $0 < s \leqslant (x-1)/2$, $x > 1$,

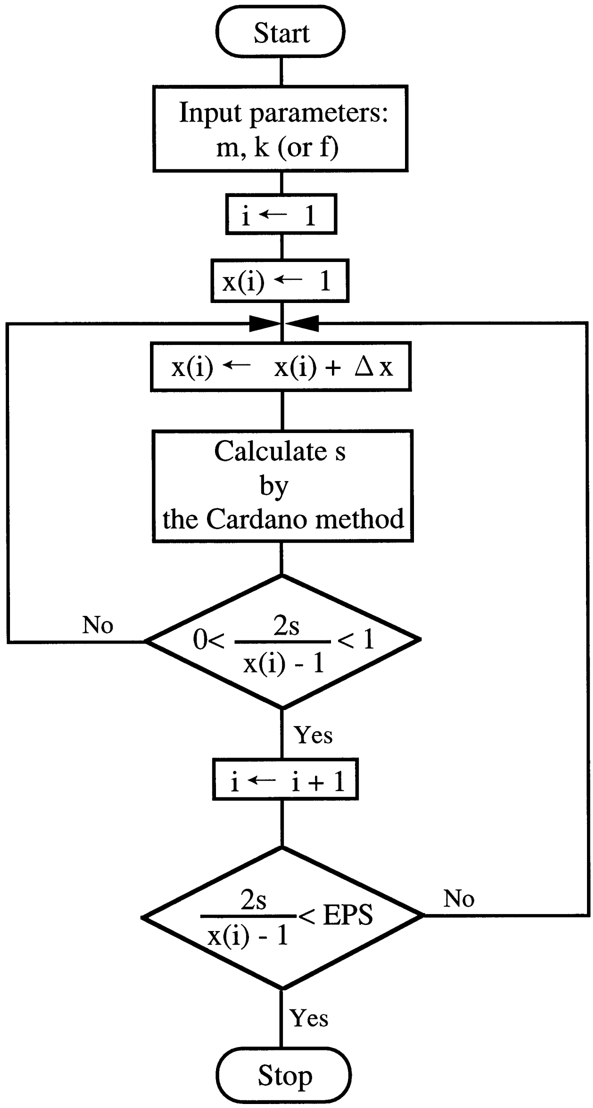

Figure 2. Flowchart of the computer program solving parameter (s).

$$d = \frac{m(x^4 - 1) - k}{4(1 - m)(1 + x)}, \qquad k = \frac{4D}{\pi R_0^4 E_f}. \tag{17}$$

The parameter k is defined as relative flexural rigidity.

Equation (16) is about the function of s, and there exits only one real number solution as assessed by the Cardano method. Substituting the real number solution in equation (15) for the cost expression shown in equation (13), the curves of the relative costs varying with the thickness ratio can easily be calculated. Figure 2 shows a flowchart of the computer program solving the parameter (s) in a different given value x.

3.2. Optimal design problem with respect to bending strength

Considering the condition of a given bending strength, the same expression as in equation (16) can be obtained. However, the parameter (k) in equation (17) is written as:

$$k = f(x - 1), \tag{18}$$

where f is the relative bending strength, defined as

$$f = \frac{2M}{\pi R_0^3[\sigma]}. \tag{19}$$

4. DISCUSSION

4.1. Effect of the thickness ratio on flexural rigidity

In order to make effective use of the properties of the materials used in a sandwich pipe. The core material is usually chosen to be of low density or low cost $\alpha_f > \alpha_c$. For this kind of sandwich pipe construction, the skin modulus is usually much higher than the core modulus, and the core thickness is also much larger than the skin thickness. Therefore, the rigidity ratio (m) approaches zero, and the thickness ratio (λ) is considerably lower than 1.

If the weight or the cost of a sandwich pipe is a certain value, a choice of weight or cost allocation between skins and core (the thickness ratio λ) can be given a maximum flexural rigidity. Figures 3 and 4 show the variation of the flexural rigidity with the thickness ratio (λ). Figure 3 gives the results of variation in the modulus ratio (m) when parameters $\alpha_c = 0.02$ and $\alpha_f = 0.2$. The results varying parameter α_c are given in Fig. 4 when the modulus ratio $m = 0.04$ and $\alpha_f = 0.2$.

The optimal values for the rigidity of sandwich pipes can be obtained when the thickness ratio (λ) is within the range of less than 10%. It is important that the thickness ratio (λ) is chosen to optimize the material properties of sandwich pipes. Especially, the choice of the thickness ratio (λ) becomes more important when the core layer has a higher modulus. This is because the flexural rigidity varies rapidly with the thickness ratio (λ).

It can be seen from Fig. 3 that the maximum point will disappear with increase in the core modulus. The flexural rigidity for the sandwich pipe becomes the maximum value when all of the core material is used. This is because core material with a high modulus can contribute to bending resistance more than the skin material.

It is shown in Fig. 4 that the flexural rigidity becomes smaller when the efficiency α_c is increased. At the end, the maximum point will disappear with increasing α_c. The flexural rigidity for the sandwich pipe reaches a maximum value when the whole skin material is used. It is clear that the core density, the cost, or both become so large that the core material cannot contribute to bending resistance.

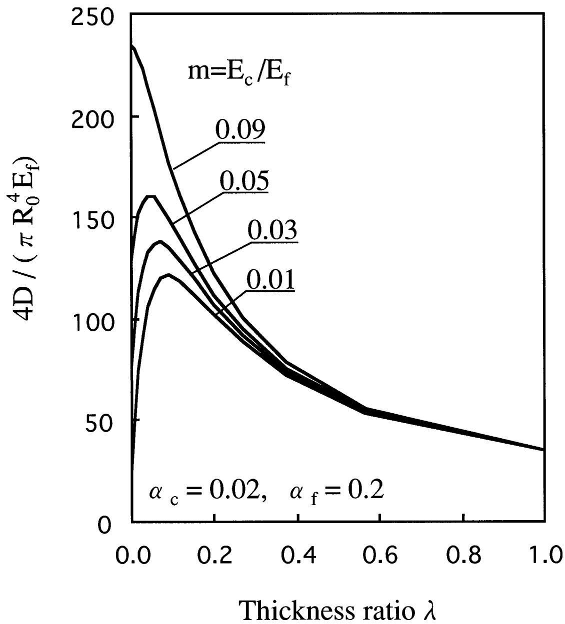

Figure 3. Effect of the thickness ratio on the flexure rigidity when varying the modulus ratio.

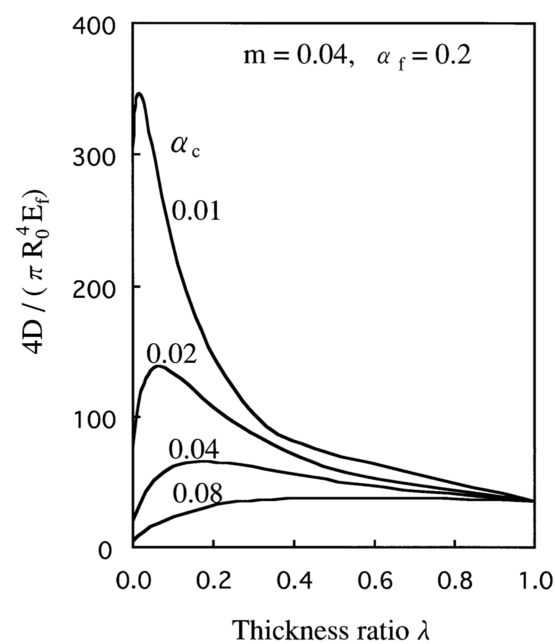

Figure 4. Effect of the thickness ratio on the flexure rigidity when varying the parameter α_c.

4.2. Effect of the thickness ratio on bending strength

Figures 5 and 6 show the variation of bending strength with the thickness ratio. Similar curve distributions are obtained with the flexural rigidity analysis. The

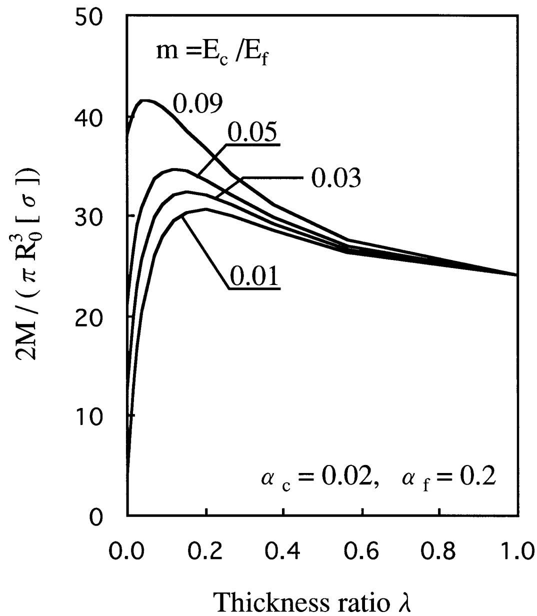

Figure 5. Effect of the thickness ratio on the flexure strength when varying the modulus ratio.

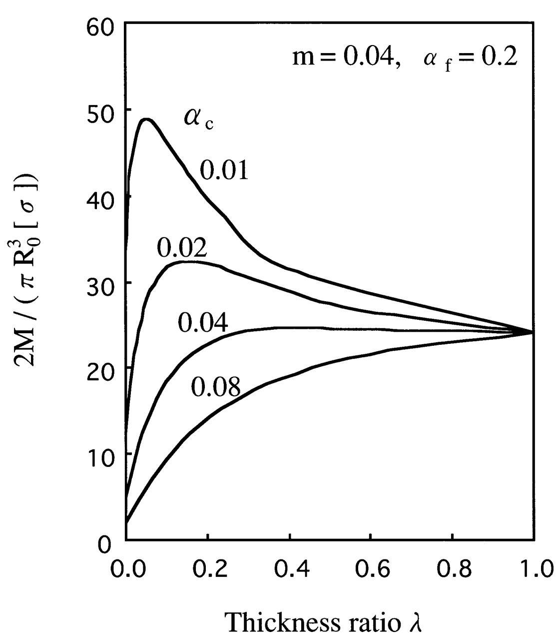

Figure 6. Effect of the thickness ratio on the flexure strength when varying the parameter α_c.

maximum bending strength occurs when the thickness ratio is near 0.1. The maximum point will move toward thinner skin thickness with increase in the core modulus or decrease of the parameter α_c (core density or cost or both).

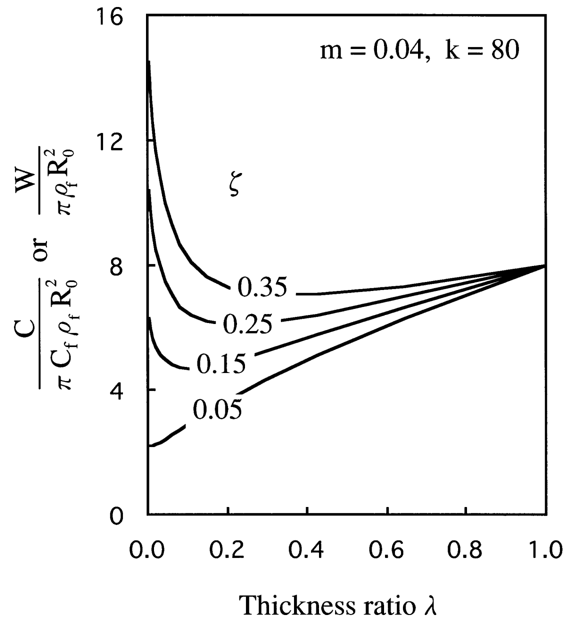

Figure 7. Relationship between the cost (or weight) of a given relative flexural rigidity and thickness ratio when varying the parameter ζ.

4.3. Effect of the thickness ratio on the cost of a given flexural rigidity

Figures 7 and 8 show the relationship between the relative cost (or the relative weight) and the thickness ratio (λ) when a flexural rigidity $k = 80$ is given. The optimum costs can be obtained in some thickness ratio.

Figure 7 gives the relative cost (or the relative weight) curves varying the parameter ζ when the modulus ratio $m = 0.04$. No optimum value exists if the costs (or the weights) of the core and skin materials are very different in a sandwich pipe construction. A physical interpretation can be made, namely, the cost (or weight) of a product having a given flexural rigidity can be decided by lowering the material cost.

Figure 8 gives the relative cost (or the relative weight) curves varying the modulus ratio (m) when the parameter $\zeta = 0.15$. The relative cost (or weight) of a given flexural rigidity decreases with increasing the modulus ratio (m). This is because core material with lower cost (or weight) can be largely used instead of skin material with higher cost (or weight) on condition that a given flexural rigidity is sufficient.

4.4. Effect of the thickness ratio on the cost of a given bending strength

Figures 9 and 10 show the relationship between the cost of a bending strength and thickness ratio when the relative bending strength $f = 30$. The relative cost (or weight) curves of a given bending strength have the same variation as those of a given flexural rigidity.

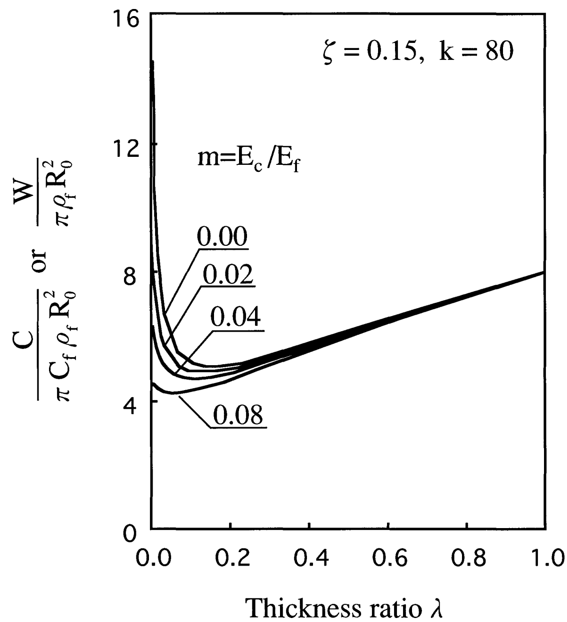

Figure 8. Relationship between the cost (or weight) of a given relative flexural rigidity and thickness ratio when varying the modulus ratio.

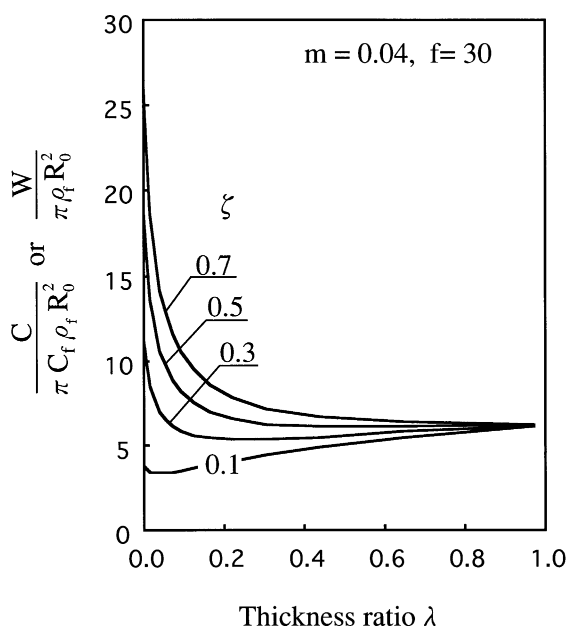

Figure 9. Relationship between the cost (or weight) of a given relative strength rigidity and thickness ratio when varying the parameter ζ.

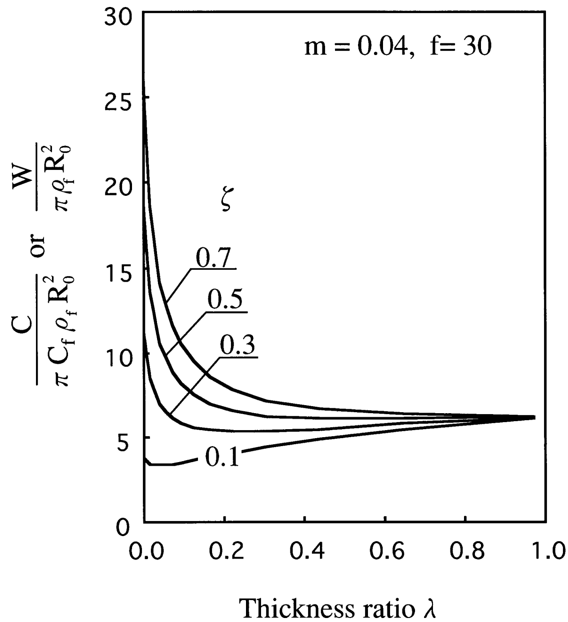

Figure 10. Relationship between the cost (or weight) of a given relative strength rigidity and thickness ratio when varying the modulus ratio.

5. CONCLUSIONS

In this paper, an optimum method for a sandwich pipe is presented for optimum sandwich pipe construction design. The optimum of the sandwich pipe is given under a few conditions: optimal design of flexural stiffness and bending strength with regard to a certain weight or cost, and minimum weight and cost designs with regard to a certain flexural stiffness or bending strength. Moreover, the effect of the design parameters on the flexural rigidity and bending strength of the sandwich pipe are discussed. Weight and cost savings can be achieved by carefully selecting the parameters and geometry of the sandwich pipe.

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